

# ISOVECTOR VIBRATIONS IN NUCLEAR MATTER AT FINITE TEMPERATURE

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## Abstract

We consider the propagation and damping of isovector excitations in heated nuclear matter within the Landau Fermi-liquid theory. Results obtained for nuclear matter are applied to calculate the Giant Dipole Resonance (GDR) at finite temperature in heavy spherical nuclei within Steinwedel and Jensen model.

The centroid energy of the GDR slightly decreases with increasing temperature and the width increases as  $T^2$  for temperatures  $T < 5$  MeV in agreement with recent experimental data for GDR in  $^{208}\text{Pb}$  and  $^{120}\text{Sn}$ .

The validity of the method for other Fermi fluids is finally suggested.

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In recent years the GDR built on highly excited states is in the center of many experimental and theoretical studies (c.f. [1] and references therein). In this context, one of the most important open problems is the behaviour of the GDR width in *nonrotating* nuclei as a function of temperature. There are two essentially different theoretical approaches to this problem. The first one [2] explains the temperature increasing of the width as an effect of the adiabatic coupling of the GDR to thermal shape deformations. In the second approach [3, 4, 5] the thermal contribution to the damping width arises from

an increasing nucleon-nucleon collision rate ( $2p2h$  excitations) plus a Landau spreading due to thermally allowed  $ph$  transitions [6, 7, 8, 9].

In the present work, following the ideology of the second approach, we consider isovector volume vibrations in spin-isospin symmetrical nuclear matter at finite temperature. A similar problem was considered in Refs. [7, 8] within the RPA method. However the Landau damping mechanism of the dissipation of a propagating mode due to thermal smearing of Fermi distribution is too weak to be responsible for the fast increase of the observed GDR width with temperature [7, 8, 11]. This problem can be solved by taking into account the two-body dissipation through the collision integral of the Landau-Vlasov equation [3]. The use of a quantum kinetic equation leads to memory effects in the collision term in order to include off energy-shell contributions [12]. Moreover it was shown in Refs. [13, 14], that memory effects are essentially increasing the widths of multipole resonances at small temperatures. In this Rapid Communication we calculate the isovector strength function of nuclear matter taking into account both thermal Landau damping and two-body collisional dissipation, including the quantum memory contribution.

The isovector response of uniform nuclear matter is described by the linearized Landau-Vlasov equation with a collision term treated in the relaxation time approximation [4, 9, 14]

$$\frac{\partial}{\partial t}\delta f + \mathbf{v} \cdot \nabla_r \delta f - \nabla_r(\delta U + 2\delta V) \cdot \nabla_p f_{eq} = -\frac{1}{\tau}\delta f|_{l \geq 1}, \quad (1)$$

where  $\delta f \equiv \delta f_n - \delta f_p$  and  $\delta U \equiv \delta U_n - \delta U_p$  are differences between neutron and proton distribution functions (d.f.) and mean fields respectively,  $\delta V \equiv \delta V_n - \delta V_p$  is external field ( $\delta V_q = \tau_q \delta V$ ,  $\tau_n = +1$ ,  $\tau_p = -1$ ) [7],  $f_{eq}(\epsilon_p = p^2/2m)$  is the equilibrium finite temperature Fermi distribution, and the notation  $l \geq 1$  means that the perturbation of the d.f.  $\delta f|_{l \geq 1}$  in collision integral includes only Fermi surface distortions with multipolarity  $l \geq 1$  in order to conserve the particle number in collision processes [12]. The inclusion of the  $l = 1$  harmonic in the collision integral of Eq. (1), at variance with the isoscalar case [9], is due to nonconservation of the isovector current, i.e. due to a collisional friction force between counterstreaming neutron and proton flows.

The dynamical component of the isovector mean field  $\delta U$  can be expressed

in terms of the isovector Landau parameter  $F'_0$  :

$$\delta U = \frac{F'_0}{N(T)} \delta \rho , \quad (2)$$

where

$$\delta \rho(\mathbf{r}; t) = \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} \delta f(\mathbf{r}, \mathbf{p}; t) , \quad (3)$$

is the density perturbation,  $g = 2$  is the spin degeneracy factor and

$$N(T) = \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} \left( -\frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} \right) \quad (4)$$

is the thermally averaged density of states,  $N(0) = gp_F m / 2\pi^2 \hbar^3$ , where we put for simplicity  $m^* = m = 938$  MeV.

For an external field  $\delta V \propto \exp(i\mathbf{k}\mathbf{r} - i\omega t)$ , periodic in space and time, the isovector collective response function [7] can be derived from Eq. (1):

$$\chi^{\text{coll}}(\omega, \mathbf{k}) = -\frac{\delta \rho}{\delta V} = \frac{2N(T)\chi_T^\tau}{1 + F'_0 \chi_T^\tau} , \quad (5)$$

where  $\chi_T^\tau$  is the intrinsic response function [15, 16]. The explicit form of the function  $\chi_T^\tau(\omega, \mathbf{k})$  is (details of derivation in Ref. [9]):

$$\chi_T^\tau(s) = -\frac{N(0)}{mp_F N(T)} \int_0^\infty dp \frac{p^2 s \chi(\bar{p}s/p)}{s' + is'' \chi(\bar{p}s/p)} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} , \quad (6)$$

where

$$\bar{p} = p_F \left( \frac{\bar{\epsilon}}{\epsilon_F} \right)^{1/2} , \quad (7)$$

$$\bar{\epsilon} = \frac{5}{3\rho_{\text{eq}}} \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} \epsilon_p f_{\text{eq}}(\epsilon_p) , \quad (8)$$

$$\rho_{\text{eq}} = \int \frac{g d\mathbf{p}}{(2\pi\hbar)^3} f_{\text{eq}}(\epsilon_p) \quad (9)$$

are quasiparticle average momentum, average kinetic energy ( respectively normalized at  $T = 0$  on  $p_F$  and  $\epsilon_F$ ) and density, with the complex variable

$$s = s' + is'', \quad s'' = \frac{m}{\tau \bar{p} k}, \quad s' = \frac{\omega m}{\bar{p} k}, \quad (10)$$

$\chi(z)$  is a Legendre function of the second kind

$$\chi(z) = \frac{1}{2} \int_{-1}^1 d\mu \frac{\mu}{\mu - z}. \quad (11)$$

Eq. (6) for the intrinsic response function of isovector vibrations has only a minor difference with isoscalar case. Namely, to recover the isoscalar response function given by Eq. (30) in Ref. [9], one should change  $is'' \rightarrow is''(1 + 3s's\bar{p}^2/p^2)$  in the denominator of Eq. (6). This difference is just due to inclusion of the damping of the  $l = 1$  harmonic in the isovector channel. We note, that the r.h.s. of Eq. (30) in Ref. [9] has an error: it should be multiplied by a minus sign.

For a given momentum transfer  $k$ , the strength function per unit volume is:

$$S_k(\omega) = \frac{1}{\pi} \text{Im}(\chi^{coll}) = \frac{2N(T) \text{Im}(\chi_T^\tau)/\pi}{(1 + F'_0 \text{Re}(\chi_T^\tau))^2 + (F'_0 \text{Im}(\chi_T^\tau))^2}. \quad (12)$$

The strength function satisfies the following energy weighted sum rule (EWSR) [7, 8]:

$$\int_0^\infty d\omega \omega S_k(\omega) = \frac{k^2}{2m} \rho_0, \quad (13)$$

where  $\rho_0 = 0.16 \text{ fm}^{-3}$  is the nuclear saturation density.

Collective modes are given by poles of the response function (5):

$$1 + F'_0 \chi_T^\tau(s) = 0. \quad (14)$$

By solving Eq. (14) we obtain the complex frequency :

$$\omega = \omega_R + i\omega_I = k \frac{\bar{p}}{m} (s - is''). \quad (15)$$

The application of the formalism discussed above to finite nuclei is based on the Steinwedel-Jensen (SJ) picture [7, 8, 17] which describes the GDR in heavy nuclei as a volume polarization mode conserving the total density  $\rho = \rho_n + \rho_p$ . According to this model, we choose the wavenumber of the

normal mode as  $k = \pi/2R$ , where  $R$  is the radius of a nucleus. Inside the nucleus, the unperturbed distribution of nucleons is supposed to be uniform. The SJ model gives a good overall reproduction of the ground state GDR energies for heavy spherical nuclei [18].

We have to remark that the calculation of GDR widths in finite nuclei is a much more difficult problem. It can be only partially solved within our nuclear matter approach, since shell effects and the escape width due to particle emission are not taken into account in the present investigation. However in the temperature region  $T = 1 \div 3$  MeV the shell effects start to smear out and the escape width is still very small. Thus we expect our calculations to be quite reliable in this temperature region.

Eq. (1) contains two free parameters: the isovector Landau parameter  $F'_0$  and the two-body relaxation time  $\tau$ .

The Landau parameter  $F'_0$  at zero temperature can be expressed as a function of the symmetry energy coefficient  $\beta$  in the Weizsäcker mass formula as follows [19]:

$$F'_0(T = 0) = \frac{3\beta}{\epsilon_F} - 1 . \quad (16)$$

For the standard value of  $\beta = 28$  MeV we have  $F'_0(0) = 1.33$ . The value of the  $F'_0$  decreases with temperature due to the decrease of the level density  $N(T)$ :

$$F'_0(T) \simeq F'_0(0) \left[ 1 - \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F} \right)^2 \right] . \quad (17)$$

However the coupling constant  $F'_0(T)/N(T)$  is independent on temperature.

The two-body relaxation time  $\tau$  includes temperature and memory effects :

$$\tau = \frac{\hbar \alpha^{(-)}}{T^2 + (\hbar \omega_R / 2\pi)^2} . \quad (18)$$

The dependence of the relaxation time on the frequency  $\omega_R$  arises from memory effects and corresponds to the Landau prescription [12]. The coefficient  $\alpha^{(-)}$  depends on nucleon-nucleon scattering cross sections. We have calculated this coefficient using energy and angular dependent differential cross sections of  $pp$  and  $np$  scattering derived from Bonn A potential both with and without in-medium corrections [20, 21] (see Appendix). Results are :  $\alpha^{(-)} = 2.3$  (5.5) MeV in the case of vacuum (in-medium reduced) cross sections.

Fig. 1 shows the photoabsorption cross section by a thermally excited nucleus  $^{208}\text{Pb}$ , which can be expressed in terms of the strength function (12) as follows:

$$\sigma_{abs}(\hbar\omega) = \frac{4\pi^2 e^2}{\hbar c k^2 \rho_0} \frac{NZ}{A} \hbar\omega S_k(\omega) . \quad (19)$$

Expression (19) is obtained from comparison of the EWSR (13) and the dipole sum rule [17]

$$\int_0^\infty dE \sigma_{abs}(E) = \frac{2\pi^2 e^2 \hbar}{mc} \frac{NZ}{A} . \quad (20)$$

As the temperature is growing, the centroid energy of the GDR (i.e. the peak of the photoabsorption cross section) is slightly shifting to the left and the width is increasing. At variance with pure mean field predictions [7, 8, 9], we see a shift to low frequencies due to the collision integral in Eq. (1) (c.f. Refs. [22, 23, 24]). Indeed at larger temperatures an increasing two-body dissipation should reduce the frequency of the collective motion, in close analogy with a classical oscillator with a friction force.

In Fig. 2a (solid lines) we report the temperature dependence of the full width half maximum (FWHM) and the centroid energy  $E_{GDR}$  as functions of temperature, calculated from the photoabsorption cross section (19) for  $^{208}\text{Pb}$ . In parallel, in Fig. 3 (solid lines) the real and imaginary parts of the pole of the response function (5) are shown. As far as the collective mode is underdamped, i.e.  $|\text{Im}(\omega)|/|\text{Re}(\omega)| \ll 1$ , an approximate relation

$$\text{FWHM} = 2 |\text{Im}(\omega)| \quad (21)$$

has to be fulfilled [14]. We see from comparison of Figs. 2a and 3, that the condition (21) is really satisfied, corresponding to a Breit-Wigner-like shape of the photoabsorption strength (see Fig. 1). However, at higher temperatures  $T > 4$  MeV the photoabsorption strength becomes much more closer to the Lorentzian shape [25].

At low temperatures, one can neglect the temperature spreading of the equilibrium Fermi distribution substituting  $\delta(\epsilon_F - \epsilon_p)$  instead of  $(-\partial f_{eq}(\epsilon_p)/\partial \epsilon_p)$  into Eq. (6). Thus we obtain the following approximate low temperature expression for the intrinsic response function :

$$\chi_T^\tau(s) \simeq \frac{s\chi(s)}{s' + is''\chi(s)} , \quad (22)$$

where variables  $s$ ,  $s''$  and  $s'$  are defined by Eqs.(10) with the change  $\bar{p} \rightarrow p_F$ . We remark that Eq.(22) when applied to the case of an electron gas ( $F_0 = N(T) 4\pi e^2/k^2$ ) gives just the longitudinal dielectric function

$$\epsilon(\mathbf{k}, \omega) = 1 + F_0 \chi_T^\tau \quad (23)$$

obtained by Mermin [26]. This is to stress the more general framework of the results presented here [27].

In the rare collision regime ( $\omega_R \tau \gg 1$ ), an approximate solution of the dispersion relation (14) with the intrinsic response function Eq.(22) can be found analytically (c.f. Refs. [12, 28]) :

$$\omega_R \simeq v_F k s^{(0)} + O(T^4) , \quad (24)$$

$$\omega_I \simeq \frac{(2F'_0 + 1)((s^{(0)})^2 - 1) - (F'_0)^2}{\tau F'_0(F'_0 - (s^{(0)})^2 + 1)} + O(T^6) , \quad (25)$$

where  $s^{(0)}$  is the root of collisionless dispersion relation

$$1 + F'_0 \chi(s^{(0)}) = 0 . \quad (26)$$

The Landau parameter  $F'_0$  in Eqs.(25), (26) is taken at  $T = 0$ . The simple expression Eq.(25) for imaginary part of the frequency  $\omega$  (long-dashed line on Fig. 3) reproduces the results of a numerical solution of the "exact" dispersion relation (14) (solid line on the same Fig. 3) with a good accuracy for temperatures  $T < 2$  MeV. At larger temperatures, a slight increase of the damping due to temperature smearing of the Fermi distribution is obtained with the dispersion relation Eq.(14). The difference between these two solutions is of the order of the Landau damping rate in the pure mean field approach [7, 8]. We see that thermal Landau damping is small for the case of GDR, at variance with the case of the breathing mode [9], since the isovector Landau parameter is larger than the isoscalar one for nuclear effective interactions at normal density. As a consequence a weaker coupling between single particle and collective motion is expected for isovector vibrations [15].

The dominance of the collisional contribution to the total damping rate of the GDR is just expressed by an approximate relation (short-dashed line on Fig. 3)

$$-\text{Im}(\omega) \simeq \frac{1}{\tau} , \quad (27)$$

which was used, for instance, in Ref. [13] to calculate widths of giant resonances. The main deviation from formula (27) in the dispersion relation Eq.(14) is caused by the exclusion of the  $l = 0$  harmonic from the collision integral in the r.h.s. of Eq.(1), i.e. due to taking into account the particle number conservation. This results in a smaller absolute value of the  $\text{Im}(\omega)$  (see Fig. 3).

As already discussed, a source of uncertainty in our calculations is given by the choice of the nucleon-nucleon cross sections. In Fig. 2 we present calculations with in-medium reduced cross sections (solid lines) and with free-space cross sections (dashed lines) in comparison with experimental widths for nuclei  $^{208}\text{Pb}$  (a) and  $^{120}\text{Sn}$  (b). There is a quite good reproduction of the experimental trend independently on the choice of cross sections.

In conclusion, we have studied the isovector response of heated spin-isospin symmetric nuclear matter on the basis of the linearized Landau-Vlasov equation with a collision integral including memory effects in the relaxation time approximation. The contribution of the thermal Landau damping into the total damping rate of isovector vibrations is found to be very small. Thus the relaxation of the volume isovector mode is caused mainly by nucleon-nucleon collisions. Increasing temperature shifts the centroid energy of the isovector strength function to smaller values. The calculated width is proportional to  $1/\tau$  in the temperature region  $T < 5$  MeV studied in this letter, that corresponds to the collisional damping of the isovector zero sound mode. This leads to a  $T^2$  behaviour of the  $GDR - FWHM$  in the region where the collective mode can propagate.

We have shown that some general GDR properties at high excitation energy can be obtained directly from Fermi liquid theory. However the aim of this work is not to get a perfect agreement with data for finite nuclei particularly at low temperatures. Indeed we are well aware that other contributions to the damping are missing in the present approach: (i) The fragmentation width, observed in RPA calculations, which can be interpreted as a Landau damping mechanism in finite Fermi systems present also at zero temperature [31]. (ii) Thermal shape fluctuations [2, 39]. (iii) Fluctuations due to nucleon-nucleon correlations [33].

A recent statistical model analysis of  $\gamma$ -spectra produced by inelastic  $\alpha$ -scattering on  $^{120}\text{Sn}$  [25] resulted in conclusion that neither the thermal fluctuation model of Ref. [39] nor the collisional damping model of this work could reproduce data in details. We believe that the combination of the two



models would give a much better agreement.

Extension to isospin asymmetric nuclear systems, more suitable for the *Pb* case, can be performed following the approach of ref.s [34, 35, 40]. We do not expect to have substantial variations in the temperature behaviour of the isovector and isoscalar modes unless very high charge asymmetries are reached [34, 35]. However in charge asymmetric nuclei, new soft mode, different from isovector and isoscalar ones, appears [40] due to collisional coupling of proton and neutron vibrations, that requires further investigations.

Finally a comment on the validity of the formalism applied here to the study of hot *GDR* in nuclei to a broader context of physical problems. Indeed it can be easily generalized to the description of relative vibrations of any two-component Fermi liquid with a mutual attraction, for instance of a Coulomb plasma consisting of opposite charged fermions. Another application could be to the oscillations of the electronic cloud in metallic clusters, where momentum nonconserving  $l = 1$  term in collision integral appears due to scattering of electrons on impurities [26].

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## Appendix

Here we will calculate the coefficient  $\alpha^{(-)}$  in Eq. (18) for the collisional relaxation time  $\tau$ . For simplicity, we will use in derivation the Boltzmann-Uehling-Uehlenbeck (BUU) collision integrals without memory effects. However, as it is shown in Ref. ([36]), the collision integrals with memory effects give the result which can be obtained using Landau prescription  $\tau = \tau_{BUU}/(1 + (\hbar\omega_R/2\pi T)^2)$ , where  $\tau_{BUU} = \hbar\alpha^{(-)}/T^2$  is the relaxation time given by BUU collision integrals.

Time evolution of the space-uniform isovector d.f.  $f = f_n - f_p$  satisfies the equation ([13, 14]) :

$$\frac{\partial f(\mathbf{p}, t)}{\partial t} = I = I_{nn} + I_{np} - I_{pp} - I_{pn} , \quad (28)$$

where  $I_{q_1 q_2}$  stands for the collision integral of particles of the sort  $q_1 = n, p$  with particles of the sort  $q_2 = n, p$ . Explicitly :

$$I_{q_1 q_2}(\mathbf{p}_1, t) = \frac{4}{(2\pi\hbar)^6} \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 w_{q_1 q_2}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_3, \mathbf{p}_4) \delta(\Delta\epsilon) \delta(\Delta\mathbf{p}) \\ Q(f_{q_1}(\mathbf{p}_1), f_{q_2}(\mathbf{p}_2); f_{q_1}(\mathbf{p}_3), f_{q_2}(\mathbf{p}_4)) , \quad (29)$$

where

$$Q(f_1, f_2; f_3, f_4) \equiv (1 - f_1)(1 - f_2)f_3f_4 - f_1f_2(1 - f_3)(1 - f_4) ;$$

$w_{q_1 q_2}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_3, \mathbf{p}_4)$  is the spin-averaged probability of two-body collisions with initial momenta  $(\mathbf{p}_1, \mathbf{p}_2)$  and final momenta  $(\mathbf{p}_3, \mathbf{p}_4)$ ;  $\Delta\epsilon = \epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4$ ,  $\epsilon_i = p_i^2/2m$ ,  $i = 1, 2, 3, 4$ ,  $\Delta\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4$ . Neglecting the dependence of  $w_{q_1 q_2}$  on the d.f., we can write down perturbations of collision integrals (29) keeping terms of the first order in  $\delta f_q$  :

$$\delta I_{q_1 q_2}(\mathbf{p}_1, t) = \frac{4}{(2\pi\hbar)^6} \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 w_{q_1 q_2}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_3, \mathbf{p}_4) \delta(\Delta\mathbf{p}) \\ \{ \alpha^{(1)} \psi_{q_1}(\mathbf{p}_1) + \alpha^{(2)} \psi_{q_2}(\mathbf{p}_2) + \alpha^{(3)} \psi_{q_1}(\mathbf{p}_3) + \alpha^{(4)} \psi_{q_2}(\mathbf{p}_4) \} , \quad (30)$$

where

$$\psi_q(\mathbf{p}) \equiv \delta f_q(\mathbf{p}) \left( \frac{\partial f_{eq}(\epsilon)}{\partial \epsilon} \right)^{-1} , \quad (q = n, p) , \quad (31)$$

$$\alpha^{(i)} \equiv \frac{\delta Q(f_{eq}(\epsilon_1), f_{eq}(\epsilon_2); f_{eq}(\epsilon_3), f_{eq}(\epsilon_4))}{\delta f_{eq}(\epsilon_i)} \\ \frac{\partial f_{eq}(\epsilon_i)}{\partial \epsilon_i} \delta(\Delta\epsilon) , \quad i = 1, 2, 3, 4 . \quad (32)$$

In Eq. (30) the isospin-symmetric nuclear matter is considered that results in the same equilibrium d.f. for neutrons and protons.

For the perturbation of the collision integral  $I$  of the r.h.s. of Eq. (28), assuming isotopic invariance ( $w_{pp} = w_{nn}$ ,  $w_{pn} = w_{np}$ ) we can write after simple algebra :

$$\begin{aligned} \delta I = & \delta I_{nn} + \delta I_{np} - \delta I_{pp} - \delta I_{pn} = \\ & \frac{4}{(2\pi\hbar)^6} \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 \delta(\Delta\mathbf{p}) \{ (w_{pp} + w_{np}) (\alpha^{(1)} \psi(\mathbf{p}_1) + \alpha^{(3)} \psi(\mathbf{p}_3)) + \\ & (w_{pp} - w_{np}) (\alpha^{(2)} \psi(\mathbf{p}_2) + \alpha^{(4)} \psi(\mathbf{p}_4)) \} , \end{aligned} \quad (33)$$

where  $\psi(\mathbf{p}) = \psi_n(\mathbf{p}) - \psi_p(\mathbf{p})$ .

The triple integral over momenta in (33) can be taken using Abrikosov-Khalatnikov transformation [12], which is valid in the limit  $T \ll \epsilon_F$  :

$$\int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 \delta(\Delta\mathbf{p}) = \frac{(m^*)^3}{2} \int_0^\pi d\Theta \frac{\sin \Theta}{\cos(\Theta/2)} \int_0^\pi d\phi \int_0^{2\pi} d\phi_2 \int_0^\infty d\epsilon_2 d\epsilon_3 d\epsilon_4 , \quad (34)$$

where  $\theta = (\hat{p}_1, \hat{p}_2)$  is the angle between momenta of colliding particles ( $\hat{p}_i \equiv \mathbf{p}_i/|\mathbf{p}_i|$ ,  $i = 1, 2, 3, 4$ );  $\phi$  is the angle between planes given by momenta of incoming and outgoing particles :

$$\cos \phi = \frac{[\hat{p}_1 \times \hat{p}_2] \cdot [\hat{p}_3 \times \hat{p}_4]}{||[\hat{p}_1 \times \hat{p}_2]|| ||[\hat{p}_3 \times \hat{p}_4]||} ; \quad (35)$$

$\phi_2$  is azimuthal angle of  $\mathbf{p}_2$  in the system with z-axis along  $\mathbf{p}_1$ .

We decompose the perturbation of the d.f. into spherical harmonics :

$$\psi(\mathbf{p}_i, t) = \sum_{l,m} \alpha_{lm}(p_i, t) Y_{lm}(\hat{p}_i) , \quad i = 1, 2, 3, 4 , \quad (36)$$

where coefficients  $\alpha_{lm}$  can be taken on the Fermi surface, since  $T \ll \epsilon_F$ .

According to Refs. [13, 28, 37], the partial relaxation time  $\tau_l$  is defined as follows :

$$\begin{aligned} \frac{1}{\tau_l^{BUU}} = & - \frac{\int_0^\infty d\epsilon \int d\Omega_{\hat{p}} Y_{lm}^*(\hat{p}) \delta I(\mathbf{p}, t)}{\int_0^\infty d\epsilon \int d\Omega_{\hat{p}} Y_{lm}^*(\hat{p}) \delta f(\mathbf{p}, t)} = \\ & \frac{\int d\Omega_{\hat{p}} Y_{lm}^*(\hat{p}) \overline{\delta I}(\hat{p}, t)}{\alpha_{lm}} , \end{aligned} \quad (37)$$

where

$$\overline{\delta I}(\hat{p}) = \int_0^\infty d\epsilon \delta I(\mathbf{p}) , \quad \epsilon = p^2/2m . \quad (38)$$

Using (34), after somewhat lengthy but standard calculations, we come to the expression :

$$\frac{1}{\tau_l^{BUU}} = \frac{(m^*)^3 T^2}{12\pi^2 \hbar^6} \left\{ \langle w_{pp} \Phi_l^{(+)} \rangle + 2 \langle w_{np} \Phi_l^{(-)} \rangle \right\} \equiv \frac{T^2}{\kappa_l} , \quad (39)$$

where angular brackets denote the averaging over angles  $\theta$  and  $\phi$  [14, 28] :

$$\langle F(\theta, \phi) \rangle \equiv \frac{1}{2\pi} \int_0^\pi d\theta \frac{\sin \theta}{\cos \frac{\theta}{2}} \int_0^\pi d\phi F(\theta, \phi) ; \quad (40)$$

$$\Phi_l^{(+)} = 1 + P_l(\hat{p}_2 \cdot \hat{p}_1) - P_l(\hat{p}_3 \cdot \hat{p}_1) - P_l(\hat{p}_4 \cdot \hat{p}_1) , \quad (41)$$

$$\Phi_l^{(-)} = 1 - P_l(\hat{p}_2 \cdot \hat{p}_1) - P_l(\hat{p}_3 \cdot \hat{p}_1) + P_l(\hat{p}_4 \cdot \hat{p}_1) . \quad (42)$$

A factor 2 at the second term in curly brackets of (39) is due to half momentum space integration over  $d\mathbf{p}_3$  in the l.h.s. of Eq. (34) [12, 38]. The arguments of the Legendre polynomials in functions (41),(42) are :

$$\begin{aligned} \hat{p}_2 \cdot \hat{p}_1 &= \cos \theta , \\ \hat{p}_3 \cdot \hat{p}_1 &= \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \cos \phi , \\ \hat{p}_4 \cdot \hat{p}_1 &= \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \cos \phi . \end{aligned}$$

For  $l = 1, 2$  and  $\infty$  we have :

$$\Phi_1^{(+)} = 0 , \quad \Phi_1^{(-)} = 4 \sin^2 \frac{\theta}{2} \sin^2 \frac{\phi}{2} , \quad (43)$$

$$\Phi_2^{(+)} = 3 \sin^4 \frac{\theta}{2} \sin^2 \phi , \quad \Phi_2^{(-)} = 3 \sin^2 \theta \sin^2 \frac{\phi}{2} , \quad (44)$$

$$\Phi_\infty^{(+)} = \Phi_\infty^{(-)} = 1 . \quad (45)$$

Collision probabilities can be expressed in terms of cross sections as follows :

$$w_{pp} = \frac{(2\pi\hbar)^3}{2\mu^2} \frac{d\sigma_{pp}}{d\Omega_{c.m.}}, \quad (46)$$

$$w_{pn} = \frac{(2\pi\hbar)^3}{2\mu^2} \frac{d\sigma_{pn}}{d\Omega_{c.m.}}, \quad (47)$$

where  $d\sigma_{pp}/d\Omega_{c.m.}$  and  $d\sigma_{pn}/d\Omega_{c.m.}$  are differential cross sections of pp and np scattering;  $d\Omega_{c.m.} = \sin\theta_{c.m.}d\theta_{c.m.}d\phi_{c.m.}$ ;  $\theta_{c.m.}$  and  $\phi_{c.m.}$  are polar and azimuthal scattering angles in the center of mass system of colliding particles;  $\mu = m/2$  is the reduced mass. Differential cross sections  $d\sigma_{pp}/d\Omega_{c.m.}$  and  $d\sigma_{pn}/d\Omega_{c.m.}$  depend on the relative momentum  $p' = |\mathbf{p}_1 - \mathbf{p}_2|/2$  of scattered particles and on the polar angle

$$\theta_{c.m.} = \arccos \left( \frac{(\mathbf{p}_1 - \mathbf{p}_2) \cdot (\mathbf{p}_3 - \mathbf{p}_4)}{|\mathbf{p}_1 - \mathbf{p}_2||\mathbf{p}_3 - \mathbf{p}_4|} \right).$$

For particles scattered on the Fermi surface, we have :

$$p' = p_F \sin \frac{\theta}{2}, \quad \theta_{c.m.} = \phi.$$

In a particular case of isotropic energy independent cross sections the result of Ref. [13] is recovered :

$$\frac{1}{\tau_1^{BUU}} = \frac{32}{9} \frac{m\sigma_v}{\hbar^3} T^2, \quad (48)$$

$$\frac{1}{\tau_2^{BUU}} = \frac{32}{15} \frac{m\sigma_s}{\hbar^3} T^2, \quad (49)$$

where  $\sigma_v = \sigma_{np}/2$ ,  $\sigma_s = (\sigma_{nn} + \sigma_{pp} + 2\sigma_{np})/4$ ,  $\sigma_{np} = (4\pi)d\sigma_{np}/d\Omega_{c.m.} \simeq 50$  mb,  $\sigma_{nn} \simeq \sigma_{pp} = (2\pi)d\sigma_{pp}/d\Omega_{c.m.} \simeq 25$  mb.

We derived relaxation times  $\tau_1^{BUU}$ ,  $\tau_2^{BUU}$  and  $\tau_\infty^{BUU}$  using pp and nn energy and angular dependent differential cross sections calculated with Bonn A potential [20, 21]. Results of these calculations both with vacuum and in-medium cross sections at normal nuclear density are given in the Table. It is seen from the Table that always  $\tau_1^{BUU} \simeq \tau_2^{BUU} \simeq \tau_\infty^{BUU}$ . This gives an idea to put the same value for all relaxation times  $\tau_l^{BUU}$ , i.e. to apply usual

relaxation time approximation. Thus, we obtain the collision integral of Eq. (1) with relaxation time  $\tau$  given by Eq. (18), where

$$\alpha^{(-)} = \left[ \frac{\hbar}{3} (\kappa_1^{-1} + \kappa_2^{-1} + \kappa_\infty^{-1}) \right]^{-1} = 2.3 \text{ (5.4) MeV}$$

for vacuum (in-medium) cross sections of Ref. [21].

**Table** Parameters  $\kappa_l$ ,  $l = 1, 2$  and  $\infty$  ( $\text{MeV}^2 \text{ fm}/c$ ) defined in Eq. (39) at various choices of nucleon-nucleon scattering cross sections.

NN cross sections	$\kappa_1$	$\kappa_2$	$\kappa_\infty$
Vacuum of Ref. [21]	503	491	401
In-medium at $\rho = \rho_0$ of Ref. [21]	1123	1068	1003
Isotropic energy- independent of Ref. [13]	920	1022	818

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## Figure captions

**Fig. 1** Photoabsorption cross section by an excited nucleus  $^{208}\text{Pb}$  as a function of photon energy for temperatures  $T = 0$  MeV (solid line),  $T = 2$  MeV (short-dashed line) and  $T = 4$  MeV (long-dashed line) calculated with in-medium cross sections of Ref. [21].

**Fig. 2** Centroid energy  $E_{GDR}$  (MeV) and  $FWHM$  (MeV) for the  $GDR$  mode in nuclei  $^{208}\text{Pb}$  (a) and  $^{120}\text{Sn}$  (b) as functions of temperature calculated with in-medium cross sections (solid lines) and with free space cross sections (dashed lines). Points with errorbars show the experimental widths from Ref. [29] –  $^{208}\text{Pb}$  and from Ref. [30] –  $^{120}\text{Sn}$ .

**Fig. 3** Temperature dependence of real and imaginary parts of the pole of the response function (5) – upper and lower solid lines respectively, and collisional width  $\Gamma_{coll} = 2\hbar/\tau$  – short-dashed line. The long-dashed line shows the imaginary part of pole as given by the approximate Eq.(25). All values are in MeV. In-medium cross sections are used.





